Integrating neighborhoods in the evaluation of fitness promotes cooperation in the spatial prisoner's dilemma game

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Abstract

A fundamental question of human society is the evolution of cooperation. Many previous studies explored this question via setting spatial background, where players obtain their payoffs by playing game with their nearest neighbors. Another undoubted fact is that environment plays an important role in the individual development. Inspired by these phenomena, we reconsider the definition of individual fitness which integrates the environment, denoted by the average payoff of all individual neighbors, with the traditional individual payoffs by introducing a selection parameter u. Tuning u equal to zero returns the traditional version, while increasing u bears the influence of environment. We find that considering the environment, *i.e.*, integrating neighborhoods in the evaluation of fitness, promotes cooperation. If we enhance the value of u, the invasion of defection could be resisted better. We also provide quantitative explanations and complete phase diagrams presenting the influence of environment on the evolution of cooperation. Finally, the universality of this mechanism is testified for different neighborhood sizes, different topology structures and different game models. Our work may shed a light on the emergence and persistence of cooperation in our life.

Keywords: Prisoner's Dilemma Game, Cooperation, Fitness, Environment

1. Introduction

The evolution of cooperation among unrelated individuals has become a major challenge in the biology and evolution research since the altruism and unselfish seems incompatible with Darwinian selection. Nevertheless, cooperation can be abundantly found in animal and human societies. In order to interpret these universal cooperative phenomena, evolutionary game theory has become a useful tool to study this puzzling dilemma [1, 2, 3]. Of particular renown is the prisoner's dilemma game, which as a paradigm illustrates the social conflict between altruistic and selfish behaviors and has attracted much attention in both theoretical and experimental studies [4]. In the original form of the game, there exist two players, and they have to simultaneously decide whether to cooperate (C) or to defect (D). Naturally, their payoffs depend on their decisions: they can get R (P) while mutual cooperation (defection); if a defector meets a cooperator, the former can obtain a maximum individual payoff T and the latter can only get a minimal payoff S. The ranking of the four payoffs must satisfy $T > R > P > S$ and $2R > T + S$. Since the defector always outperforms the cooperator, irrespective of the opponent's choice, two players will inevitably fall into the mutual defection state. Over the past decades, many mechanisms have been proposed to overcome this dilemma, such as kin selection [5], direct and indirect reciprocity [6, 7], voluntary participation [8] and spatial structure [9, 10]. Among these achievements, the most prominent success is obtained by the spatial extension.

In the pioneer work by Nowak and May [10], players were arranged on the vertex of a square lattice, and their payoffs were gathered from playing the game with their nearest neighbors. Then players were allowed to adopt the strategy of their neighbors, provided their payoff was higher. It was shown that the emergence and sustainment of cooperation could be greatly improved. Along this way, much effort has been given to discover new mechanisms that can sustain stable cooperation. They include the interplay between evolutionary games and network structure (regular networks [11, 12], small world networks [13, 14, 15] and scale-free networks [16, 17, 18]), environmental noise [19, 20, 21], additional noise introducing into payoffs [22, 23], teaching activity [24, 25, 26], the mobility of players [27, 28], asymmetric payoff [29, 30], differences in evolutionary time scales [31], aspiring to the fittest [32, 33]. For recent surveys of this field, one can refer to two extensive reviews [34, 35].

In the present work, we mainly focus on the research of individual fitness during the process of evolution. For the majority of previous researches, a player's fitness in the spatial structure equals the current accumulated payoff collected from its direct neighbors. Indeed, the meaning of fitness is often related to the reproduction ability [36, 37] and some works exhibiting the relationships between payoff and fitness are also presented. For example, Ohtsuki *et.al.* defined fitness as the accumulated payoff with a background noise [38]; Szolnoki *et.al.* featured fitness as mixture of accumulated payoff and normalized payoff [39]; Jia *et.al.* drew fitness as the accumulated payoff with a fluctuation coefficient [40]; and other researchers described fitness as combination of current payoff and past payoffs [41, 42, 43]. Inspired by their instructive suggestions, we propose a new method to define individual fitness where neighborhoods are considered.

As is known, environment plays an indispensable role in the real life and it can usually affect individual development. For example, people in the elite teams are more likely to learn technical ability from their excellent teammates to enhance competitiveness of themselves, animals in the strong groups are more easily to prey or resist the predators. Therefore, it is natural to take environment presented by the average payoff of all individual neighbors into account when talking about individual fitness. Here we redefine the individual fitness via integrating its current accumulated payoff with average payoff of all individual neighbors, and find that the evolution of cooperation can be remarkably promoted.

The paper is organized as follows. In the next section, we will first describe evolutionary prisoner's dilemma game and the adjusted definition of fitness. Subsequently, the main simulation results and discussions will be presented in Section 3. And the conclusion is summarized in the last section.

2. The Model

For simplicity, but without loss of generality, we consider an evolutionary prisoner's dilemma game with the rescaled payoff matrix: $T = b$, $R = 1$ and $P = S = 0$, where the parameter b $(1 < b < 2)$ represents the temptation to choose defection. Initially, cooperators and defectors are randomly distributed on a square $L \times L$ square lattice with equal probability. At each time step, player i firstly acquires its payoff P_i by playing game with its direct neighbors. Next, the payoffs P_j of all the neighbors of player i can be

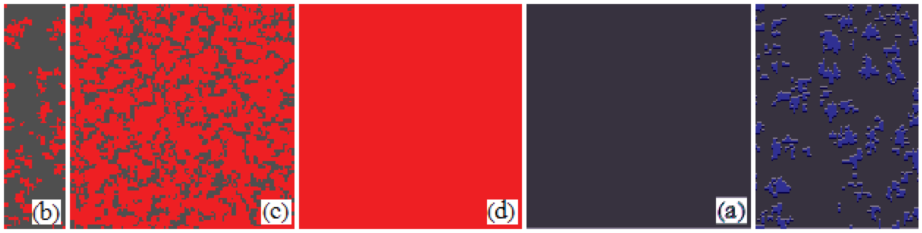

Figure 1: (*color online*) Characteristic snapshots of cooperators (red) and defectors (gray) for different values of the parameter u. From (a) to (d) $u = 0$, 0.2, 0.4 and 0.6, respectively. Depicted results were obtained for $b = 1.20$ and $K = 0.1$ on a 100×100 square lattice.

obtained by means of the same way as player i, and the environment could be characterized by the average value of all neighbors' payoffs P_j , that is,

$$\overline{P} = \frac{\sum_{j=1}^{k_i} P_j}{k_i}, \tag{1}$$

where the sum runs over all the neighbors of player i, and k_i denotes the neighbor number of player i. Then we evaluate the fitness of player i according to the following expression:

$$f_i = (1 - u) \times P_i + u \times \overline{P}, \tag{2}$$

where $0 \leq u \leq 1$ is a tunable parameter. Obviously, when $u = 0$, the fitness mathematically equals the accumulated payoff P_i, which does not consider the influence of environment and goes back to the traditional version [19, 24, 32]. Interestingly, as u increases, the environment will hold larger proportion. After each step of the game, players synchronously update their strategies. When player i is to update its strategy, it will randomly select a neighbor j and adopt its strategy with a probability $W_{(i \to j)}$ depending on the fitness difference. namely

$$W_{(i \to j)} = \frac{1}{1 + exp[(f_i - f_j)/K]}, \tag{3}$$

where $0 < K < +\infty$ characterizes the environmental noise, including irrationality and errors. The effect of noise K has been well studied in previous literatures[19, 32, 33, 46].

3. Simulation Results and Discussion

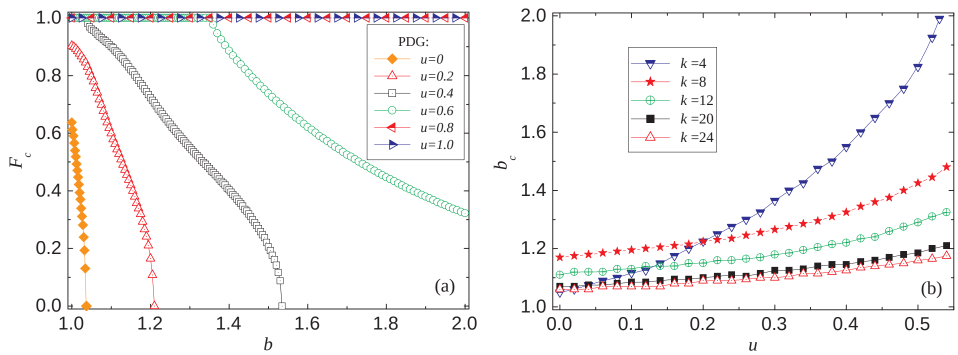

Figure 2: (*color online*) (a):Frequency of cooperators F_c in dependence on the parameter b for different values of the parameter u. Note that comparing with the traditional version (*i.e.,* $u = 0$), the cooperation can be greatly improved in our version. (b): Critical threshold values of $b = b_c$, marking extinction of cooperators, in dependence on the parameter u for different neighborhood sizes. Note that the value of b_c will monotonously increase with the the increase of u, irrespective of the neighborhood size. Due to the scale of the parameter $b(1 < b < 2)$, $b_c > 2$ is canceled. Depicted results were obtained for $K = 0.1$.

Simulations below were carried out on a 100×100 square lattice. The key quantity frequency of cooperators F_c was determined within 10^4 full Monte Carlo steps after sufficiently long transients were discarded. Moreover, each data were averaged over up to 20 independent runs for each set of parameter values in order to assure suitable accuracy.

We start by visually inspecting the spatial patterns formed by cooperators and defectors for different values of the adjusting parameter u. Figure 1 shows the results obtained by $b = 1.20$ and $K = 0.1$. For $u = 0$ (Fig.1(a)), the model returns the traditional model and the system falls into the pure-defector state when b is not very large; for $\mu = 0.2$ (Fig.1(b)), a small fraction of cooperators can sustain by forming the small yet compact clusters to resist the invasion of defectors; for $u = 0.4$ (Fig.1(c)), cooperators outperform defectors and they can form strong and large clusters to survive and prevail; for $u = 0.6$ (Fig.1(d)), the system will be entirely dominated by cooperation strategy. These results reveal that considering the environment, *i.e.*, integrating neighborhoods in the evaluation of fitness, can greatly enhance the evolution of cooperation.

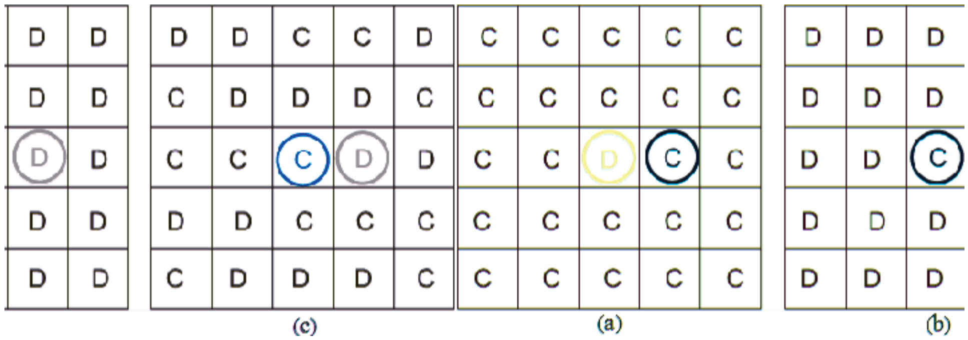

Figure 3: (*color online*) Three typical sub-patterns in the spatial game: (a) a defector surrounded by cooperators; (b) a cooperator surrounded by defectors; (c) cooperators and defectors are evenly mixed.

To quantify the effect of u more precisely, we examine the relationship between the frequency of cooperators F_c and the temptation to defect b for different values of u. As is shown in Fig.2(a), cooperation monotonously decreases with the increase of b no matter under what value of u. Interestingly, it is evident that u plays a crucial role in the evolution of cooperation: the larger the value of u, the higher the cooperation level. In particular, when u is considerably large ($u \geq 0.8$), the defectors still go extinct even if defectors possess great advantage over cooperators ($b = 2.0$), which indicates that the effect of u (*i.e.*, integrating neighborhoods in the evaluation of fitness) on facilitating cooperation is very conspicuous. Moreover, it is also worth noting how the critical value b_c, marking the extinction of cooperators, varies in dependence on the parameter u for different neighborhood sizes. Fig.2(b) demonstrates clearly the existence of an optimal neighborhood size when the value of u is small, that is, cooperation can be better promoted in an intermediate neighborhood size, which is in agreement with the result of traditional version [44]. Whereas, as the value of u increasing this trend will change, and small neighborhood size will be more beneficial for the evolution of cooperation. More importantly, we can observe that although the survival space for cooperators is becoming smaller and smaller, which could be predicted from the mean-field approximation [34, 45], the threshold value of cooperators b_c will monotonously enhance with the increasing of u, irrespective of the neighborhood sizes. This suggests that such a facilitation effect on cooperation is robust to different neighborhood sizes.

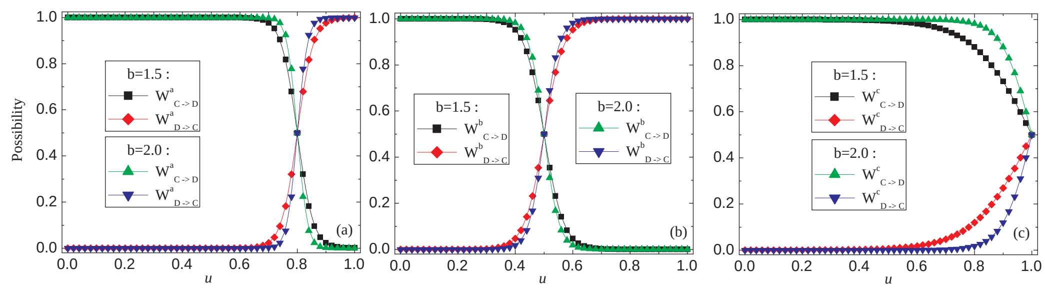

Figure 4: (*color online*) The transition possibility between the circled cooperator and defector in Fig.3. (a) corresponds to the pattern of Fig.3(a); (b) corresponds to the pattern of Fig.3(b); (c) corresponds to the pattern of Fig.3(c). Depicted results were obtained for $K = 0.1$.

Next we will analyze the underlying mechanism for this kind of promotion effect. Figure 3 shows three typical sub-patterns on square lattices, pattern (a): a defector is surrounded by cooperators; pattern (b): a cooperator encircled by defectors; pattern (c): cooperators and defectors are evenly mixed. Here we will focus on the strategy transition between the circled red cooperator and the circled blue defector. Before the formal analysis, it is indispensable to predefine some useful parameters. We assume that f_C^a, f_C^b, f_C^c (f_D^a, f_D^b, f_D^c) are the fitness of the circled cooperator (defector) in pattern (a), pattern (b), pattern (c), respectively; $\Delta_a, \Delta_b, \Delta_c$ are the fitness difference of the circled cooperator and the circled defector in pattern (a), pattern (b), pattern (c), respectively; and $W_{C\to D}^a, W_{C\to D}^b, W_{C\to D}^c$ ($W_{D\to C}^a, W_{D\to C}^b, W_{D\to C}^c$) are the transition possibility of the circled cooperator to become defector (the circled defector to become cooperator) in pattern (a), pattern (b), pattern (c), respectively. According to Eq.(2), we have:

$$\begin{aligned}
\Delta_a &= f_D^a - f_C^a = (4b-3) + (3.75-5b)u \\
\Delta_b &= f_D^b - f_C^b = b(1-2u) \qquad (4) \\
\Delta_c &= f_D^c - f_C^c = 2(b-1)(1-u).
\end{aligned}$$

The fitness difference of cooperators and defectors is crucial for the evolution of cooperation. Several previous works have shown that the emergence of cooperation on lattices is often induced by the formation of cooperator clusters, where cooperators can obtain higher payoff (fitness) to protect them-

selves against the invasion of defectors. From Eq.(4), we can find $\Delta_a, \Delta_b, \Delta_c$ monotonously decrease with the increase of u, which indicates large value of u can weaken the advantage of defectors, and will be greatly beneficial for the evolution of cooperation. Additionally, Figure 4 clearly demonstrates the transition possibility of the circled cooperator and circled defector when varying value of b for three sub-patterns. One can observe that, although the cross points of $W_{D\to C}$ and $W_{C\to D}$ are different, the result is consistent that as u increases, a defector is more likely to turn into a cooperator yet a cooperator is harder to become a defector.

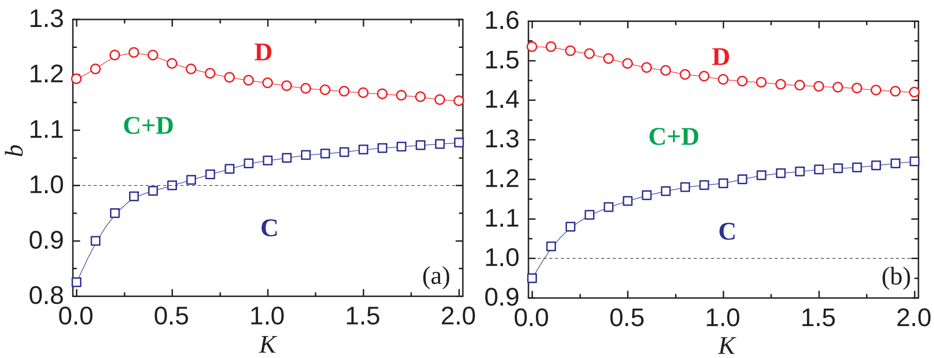

Figure 5: (*color online*)Full $b-K$ phase diagram for $u = 0.2$ (a) and $u = 0.4$ (b), obtained via Monte Carlo simulations of the prisoner's dilemma game on the square lattice. The blue and red lines mark the border between stationary pure C and D phases and the mixed C+D phase, respectively. Resembling with previous works [20, 22], it can be observed that for $u = 0.2$ (a) there exists an intermediate uncertainty in the strategy adoption process (an intermediate value of K) for which the survivability of cooperators is maximal, *i.e.*, F_c is maximal. However, the borderline separating the pure C and the mixed C+D phase for the $u = 0.4$ case (b) exhibits a monotonically decreasing curve, the D $\leftrightarrow$ C+D transition is qualitatively different.

Furthermore, we will examine how the parameter u affect the evolution of cooperation under different levels of uncertainty. The later can be tuned via K (see Eq.(3)),which acts as a temperature parameter in the employed Fermi strategy adoption function [11]. In the case of $K \to 0$, the strategy of selected neighbor is always adopted provided that its fitness is higher. While in the limit $K \to \infty$, all information is lost, and switching to neighbor's strategy is like tossing a coin. For accurately solving the problem, we obtain the full $b-K$ phase diagrams for some characteristic values of u on the square lattice.

The phase diagram illustrated in Fig.5(a) is well-known, and implies the existence of an optimal level of uncertainty for the evolution cooperation, as was previously reported in [20, 22]. In particular, note the the D $\leftrightarrow$ C+D transition line is bell shaped, indicating that $K \approx 0.3$ is the optimal temperature where cooperators are able to survive at the highest value of b. This phenomenon as the evolution resonance can only be observed on the interaction topologies lacking the overlapping triangles [19]. On the other hand, comparing to the traditional phase diagram [20, 22, 32], *i.e.* $u = 0$, the survival space of mixed strategies C+D is enhanced greatly, which will be helpful for the emergence of cooperation. Interestingly, the increasing of u completely eradicates (as do interaction networks incorporating overlapping triangles) the existence of an optimal K, as presented in Fig.5(b). The bell shaped D $\leftrightarrow$ C+D transition line gives way to a monotonously decreasing line, indicating the enhancement of uncertainty level will directly accelerate the dying out of cooperators. This quantitative change in the phase diagram implies that increasing the tunable parameter u or the proportion of environment in the individual fitness will affectively alter the interaction networks. Though the square lattice evidently lacks the overlapping triangles, which guarantees the existence of an optimal K for the small values of u, considering neighborhoods in the fitness seems to alter the likelihood and enables the linkage of essentially disconnected triplets realized. A similar report was recently investigated in public goods games, in which the joint membership in large groups was also found to alter the effective interaction network and thus the impact of uncertainly on the evolution of cooperation [46].

Lastly, it remains of interest to explore the generalization of cooperation promotion for such a new mechanism, we investigate the systematic cooperative behaviors under different complex networks and different game models (Fig.6). Similarly as Figure 2, it can be observed that the cooperation level can be greatly promoted with the increzse of u for prisoner's dilemma game on the small world network and the random regular graph. At the same time, it shows that the evolution of cooperation can be effectively guaranteed for snowdrift game and public goods game on square lattices. The result is specially inspiring in snowdrift game, since cooperation is usually thought to be inhibited by the spatial structure [36].

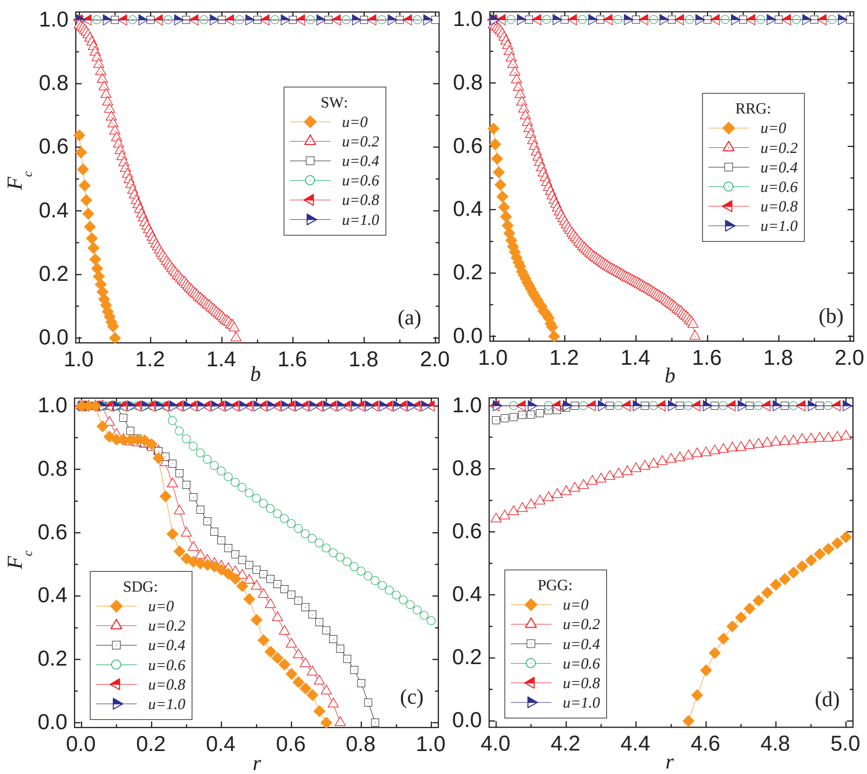

Figure 6: (*color online*) Frequency of cooperators F_{c} in dependence on the parameter b (or r) for different values of the parameter u for different complex networks and different game models: (a) prisoner's dilemma game on the Watts-Strogatz small world network (SW) with the fraction of rewired links equalling 0.1; (b) prisoner's dilemma game on the random regular graph (RRG); (c) snowdrift game on the regular lattice; (d) public goods games on the regular lattice. Note that these results are in qualitative agreement with those obtained in Fig.2(a) in that the larger value of u greatly facilitates cooperation. Depicted results were obtained for $K = 0.1$.

4. Conclusion

In summary, we have investigated the effect of redefined fitness on the cooperative behaviors whin the framework of spatial evolutionary prisoner's dilemma game. In the model, the fitness of an individual is evaluated by the combination of its current payoff and environment (the average value of all its neighbors' payoffs). It has been found that integrating neighborhoods in the evaluation of fitness can promote a remarkably high cooperation level, especially when it holds larger proportion in the fitness. Via exploring some typical sub-patterns in spatial games, we have demonstrated that cooperators considering the influence of environment can better resist the invasion of defectors.

Further interesting is the fact the increase of the parameter u, marking more proportion of environment, has altered the effective interaction network, while the similar phenomenon can be observed for public goods games in [46]. This effective transition of topology structure will provide more beneficial condition to cooperators. To prove the generality of the promotion effect, we have also examined the cooperative behaviors for different neighborhood sizes, different topology structures and different game models. It is instructive that the similar results can be observed. Our work reveals that considering environment *i.e.*, integrating neighborhoods in the evaluation of fitness, plays an important role in the evolution of cooperation, and thus it may shed a light on understanding the emergence of cooperative behaviors in natural and social systems.

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